Syllabus overview

This book covers the whole syllabus for the DP Mathematics: applications and interpretation HL course. Here is an overview of the syllabus content covered in each chapter.

1 Measuring space: accuracy and geometry

Syllabus reference	Syllabus content
SL1.5*	Laws of exponents with integer exponents. Introduction to logarithms with base 10 and e. Numerical evaluation of logarithms using technology.
SL1.1*	Introduction to logarithms with base 10 and e. Numerical evaluation of logarithms using technology.
SL1.6	Approximation: decimal places, significant figures. Upper and lower bounds of rounded numbers. Percentage errors. Estimation.
SL3.1*	The distance between two points in three-dimensional space, and their midpoint. Volume and surface area of three-dimensional solids including right-pyramid, right cone, sphere, hemisphere and combinations of these solids. The size of an angle between two intersecting lines or between a line and a plane.
SL3.2*	Use of sine, cosine and tangent ratios to find the sides and angles of right-angled triangles. The sine rule: $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$. The cosine rule $c^2=a^2+b^2-2ab\cos C$; $\cos A=\frac{b^2+c^2-a^2}{2bc}$. Area of a triangle as $\frac{1}{2}ab\sin C$.
SL3.3*	Applications of right and non-right-angled trigonometry, including Pythagoras theorem. Angles of elevation and depression. Construction of labelled diagrams from written statements.
SL3.4	The circle: length of an arc; area of a sector.
AHL1.10	Simplifying expressions, both numerically and algebraically, involving rational exponents.

AHL3.8	The definitions of $\cos\theta$ and $\sin\theta$ in terms of the unit circle. The Pythagorean identity: $\cos^2\theta + \sin^2\theta = 1$ Definition of $\tan\theta$ as $\frac{\sin\theta}{\cos\theta}$ Extension of the sine rule to the ambiguous case. Graphical methods of solving trigonometric equations in a finite interval.

**Shows content that is common to both the Mathematics: analysis and approaches and the Mathematics: applications and interpretation courses.*

2 Representing and describing data: descriptive statistics

Syllabus reference	Syllabus content
SL1.2*	Arithmetic sequences and series. Use of the formulae for the nth term and the sum of the first n terms of the sequence. Use of sigma notation for sums of arithmetic sequences. Applications. Analysis, interpretation and prediction where a model is not perfectly arithmetic in real-life.
SL4.1*	Concepts of population, sample, random sample, discrete and continuous data. Reliability of data sources and bias in sampling. Interpretation of outliers. Sampling techniques and their effectiveness.
SL4.2*	Presentation of data (discrete and continuous): frequency distributions (tables). Histograms. Cumulative frequency; cumulative frequency graphs; use to find median, quartiles, percentiles, range and interquartile range (IQR). Production and understanding of box and whisker diagrams.
SL4.3*	Measures of central tendency (mean, median and mode). Estimation of mean from grouped data. Modal class. Measures of dispersion (interquartile range, standard deviation and variance). Effect of constant changes on the original data. Quartiles of discrete data.
SL4.4*	Linear correlation of bivariate data. Pearson's product–moment correlation coefficient, r. Scatter diagrams; lines of best fit, by eye, passing through the mean point. Equation of the regression line of y on x. Use of the equation of the regression line for prediction purposes. Interpret the meaning of the parameters, a and b, in a linear regression $y = ax + b$.

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3 Dividing up space: coordinate geometry, lines, Voronoi diagrams, vectors

Syllabus reference	Syllabus content
SL2.1*	Different forms of the equation of a straight line. Gradient; intercepts Lines with gradients, m_1 and m_2 Parallel lines $m_1 = m_2$ Perpendicular lines, $m_1 \times m_2 = -1$
SL2.3*	The graph of a function; its equation $y = f(x)$. Creating a sketch from information given or a context, including transferring a graph from screen to paper. Using technology to graph functions including their sums and differences.
SL2.4*	Determine key features of graphs. Finding the point of intersection of two curves or lines using technology.
SL3.1*	The distance between two points in three-dimensional space, and their midpoint. Volume and surface area of three-dimensional solids including right-pyramid, right cone, sphere, hemisphere and combinations of these solids. The size of an angle between two intersecting lines or between a line and a plane.
SL3.5	Equations of perpendicular bisectors.
SL3.6	Voronoi diagrams; sites, vertices, edges, cells. Addition of a site to an existing Voronoi diagram. Nearest neighbour interpolation. Applications of 'the toxic waste dump' problem.
AHL3.10	Concept of a vector and a scalar. Representation of vectors using directed line segments. Unit vectors; base vectors $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$. Components of a vector; column representation: $\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \boldsymbol{v_1 i} + \boldsymbol{v_2 j} + \boldsymbol{v_3 k}$. The zero vector 0, the vector $-\boldsymbol{v}$. Position vectors $\vec{OA} = \boldsymbol{a}$ $\vec{AB} = \vec{OB} - \vec{OA} = \boldsymbol{b} - \boldsymbol{a}$ Rescaling and normalizing vectors.
AHL3.11	Vector equation of a line in two and three dimensions:

	$\boldsymbol{r} = \boldsymbol{a} + \lambda\boldsymbol{b}$, where $\boldsymbol{b}$ is a direction vector of the line.
AHL3.12	Vector applications to kinematics. Modelling linear motion with constant velocity in two and three dimensions. Motion with variable velocity in two dimensions.
AHL3.13	Definition and calculation of the scalar product of two vectors. The angle between two vectors; the acute angle between two lines. Definition and calculation of the vector product of two vectors. Geometric interpretation of $\lvert\boldsymbol{v} \times \boldsymbol{w}\rvert$. Components of vectors.

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4 Modelling constant rates of change: linear functions and regressions

<table>
<tr><th>Syllabus reference</th><th>Syllabus content</th></tr>
<tr><td>SL2.2*</td><td>Concept of a function, domain, range and graph.

Function notation, eg $f(x), v(t), C(n)$

The concept of a function as a mathematical model.

Informal concept that an inverse function reverses or undoes the effect of a function.

Inverse function as a reflection in the line $y = x$, and the notation $f^{-1}(x)$.</td></tr>
<tr><td>SL2.3*</td><td>The graph of a function; its equation $y = f(x)$.

Creating a sketch from information given or a context, including transferring a graph from screen to paper.

Using technology to graph functions including their sums and differences.</td></tr>
<tr><td>SL2.4*</td><td>Determine key features of graphs.

Finding the point of intersection of two curves or lines using technology.</td></tr>
<tr><td>SL2.5</td><td>Modelling with the following functions:
Linear models: $f(x) = mx + c$
Quadratic models: $f(x) = ax^2 + bx + c; a \neq 0$. Axis of symmetry, vertex, zeros and roots, intercepts on the x-axis and y-axis.
Exponential growth and decay models:
$f(x) = ka^x + c$
$f(x) = ka^{-x} + c$
(for $a > 0$)
$f(x) = ke^{rx} + c$

Equation of a horizontal asymptote.
Direct/inverse variation: $f(x) = ax^n, n \in \mathbb{Z}$

The y-axis as a vertical asymptote when $n < 0$.
Cubic models: $f(x) = ax^3 + bx^2 + cx + d$
Sinusoidal models: $f(x) = a\sin(bx) + d, f(x) = a\cos(bx) + d$</td></tr>
<tr><td>SL1.2*</td><td>Arithmetic sequences and series.

Use of the formulae for the nth term and the sum of the first n terms of the sequence.

Use of sigma notation for sums of arithmetic sequences.

Applications.

Analysis, interpretation and prediction where a model is not perfectly arithmetic in real-life.</td></tr>
<tr><td>SL1.8</td><td>Use technology to solve:</td></tr>
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	Systems of linear equations in up to 3 variables Polynomial equations
SL2.6	Modelling skills: • Use the modelling process described in the "mathematical modelling" section to create, fit and use the theoretical models in section SL2.5, and their graphs. Develop and fit the model: • Given a context recognize and choose an appropriate model and possible parameters. • Determine a reasonable domain for a model. • Find the parameters of a model. Test and reflect upon the model: • Comment on the appropriateness and reasonableness of a model. • Justify the choice of a particular model, based on the shape of the data, properties of the curve and/or on the context of the situation. Use the model: • Reading, interpreting and making predictions based on the model.
SL4.4*	Linear correlation of bivariate data. Pearson's product–moment correlation coefficient, r. Scatter diagrams; lines of best fit, by eye, passing through the mean point. Equation of the regression line of y on x. Use of the equation of the regression line for prediction purposes. Interpret the meaning of the parameters, a and b, in a linear regression $y = ax + b$
SL4.10	Spearman's rank correlation coefficient, r_s. Awareness of the appropriateness and limitations of Pearson's product moment correlation coefficient and Spearman's rank correlation coefficient, and the effect of outliers on each.
AHL2.7	Composite functions in context. The notation $(f \circ g)(x) = f(g(x))$ Inverse function f^{-1}, including domain restriction. Finding an inverse function.
AHL2.9	In addition to the models covered in the SL content the HL content extends this to include modelling with the following functions: • Exponential models to calculate half-life. • Natural logarithmic functions: $f(x) = a + b\ln x$ • Sinusoidal models: $f(x) = a\sin(b(x - c)) + d$

	• Logistic models: $f(x) = \frac{L}{1+Ce^{-kx}}$; $L, C, k > 0$ • Piecewise models.
AHL4.13	Non-linear regression. Evaluation of least squares regression curves using technology. Sum of square residuals (SS_{res}) as a measure of fit for a model. The coefficient of determination (R^2). Evaluation of R^2 using technology.

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5 Quantifying uncertainty: probability

Syllabus reference	Syllabus content
SL4.5*	Concepts of trial, outcome, equally likely outcomes, relative frequency, sample space (U) and event. The probability of an event A is $\mathrm{P}(A) = \frac{n(A)}{n(U)}$. The complementary events A and A' (not A). Expected number of occurrences.
SL4.6*	Use of Venn diagrams, tree diagrams, sample space diagrams and tables of outcomes to calculate probabilities. Combined events: $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$. Mutually exclusive events: $\mathrm{P}(A \cap B) = 0$ Conditional probability: $\mathrm{P}(AB) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$ Independent events: $\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B)$.
AHL4.19	Transition matrices. Powers of transition matrices. Regular Markov chains. Initial state probability matrices. Calculation of steady state and long-term probabilities by repeated multiplication of the transition matrix or by solving a system of linear equations.

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6 Modelling relationships with functions: power and polynomial functions

Syllabus reference	Syllabus content
SL2.2*	Concept of a function, domain, range and graph. Function notation, eg $f(x), v(t), C(n)$ The concept of a function as a mathematical model. Informal concept that an inverse function reverses or undoes the effect of a function. Inverse function as a reflection in the line $y = x$, and the notation $f^{-1}(x)$.
SL2.3*	The graph of a function; its equation $y = f(x)$. Creating a sketch from information given or a context, including transferring a graph from screen to paper. Using technology to graph functions including their sums and differences.
SL2.4	Determine key features of graphs. Finding the point of intersection of two curves or lines using technology.
SL2.5	Modelling with the following functions: • Linear models: $f(x) = mx + c$ • Quadratic models: $f(x) = ax^2 + bx + c; a \neq 0$. Axis of symmetry, vertex, zeros and roots, intercepts on the x-axis and y-axis. • Exponential growth and decay models: $f(x) = ka^x + c$ $f(x) = ka^{-x} + c$ (for $a > 0$) $f(x) = ke^{rx} + c$ Equation of a horizontal asymptote. • Direct/inverse variation: $f(x) = ax^n, n \in \mathbb{Z}$ The y-axis as a vertical asymptote when $n < 0$. • Cubic models: $f(x) = ax^3 + bx^2 + cx + d$ • Sinusoidal models: $f(x) = a\sin(bx) + d, f(x) = a\cos(bx) + d$
SL2.6	Modelling skills: • Use the modelling process described in the "mathematical modelling" section to create, fit and use the theoretical models in section SL2.5, and their graphs. Develop and fit the model: • Given a context recognize and choose an appropriate model and possible parameters. • Determine a reasonable domain for a model.

	• Find the parameters of a model. Test and reflect upon the model: • Comment on the appropriateness and reasonableness of a model. • Justify the choice of a particular model, based on the shape of the data, properties of the curve and/or on the context of the situation. Use the model: • Reading, interpreting and making predictions based on the model.
SL1.8	Use technology to solve: • Systems of linear equations in up to 3 variables • Polynomial equations
AHL2.7	Composite functions in context. The notation $(f \circ g)(x) = f(g(x))$ Inverse function f^{-1}, including domain restriction. Finding an inverse function.
AHL2.8	Transformations of graphs. Translations: $y = f(x) + b$; $y = f(x - a)$ Reflections: in the x-axis $y = -f(x)$, and in the y-axis $y = f(-x)$ Vertical stretch with scale factor p: $y = pf(x)$ Horizontal stretch with scale factor $\frac{1}{q}$: $y = f(qx)$ Composite transformations.
AHL2.9	In addition to the models covered in the SL content the HL content extends this to include modelling with the following functions: • Exponential models to calculate half-life. • Natural logarithmic functions: $f(x) = a + b \ln x$ • Sinusoidal models: $f(x) = a \sin(b(x - c)) + d$ • Logistic models: $f(x) = \frac{L}{1+Ce^{-kx}}$; $L, C, k > 0$ • Piecewise models.

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7 Modelling rates of change: exponential and logarithmic functions

Syllabus reference	Syllabus content
SL1.3*	Geometric sequences and series Use of the formulae for the nth term and the sum of the first n terms of the sequence. Use of sigma notation for the sums of geometric sequences. Applications.
SL1.4*	Financial applications of geometric sequences and series: • Compound interest • Annual depreciation
SL1.5*	Laws of exponents with integer exponents. Introduction to logarithms with base 10 and e. Numerical evaluation of logarithms using technology.
SL1.7	Amortization and annuities using technology.
SL2.2*	Concept of a function, domain, range and graph. Function notation, eg $f(x), v(t), C(n)$ The concept of a function as a mathematical model. Informal concept that an inverse function reverses or undoes the effect of a function. Inverse function as a reflection in the line $y = x$, and the notation $f^{-1}(x)$.
SL2.3*	The graph of a function; its equation $y = f(x)$. Creating a sketch from information given or a context, including transferring a graph from screen to paper. Using technology to graph functions including their sums and differences.
SL2.4	Determine key features of graphs. Finding the point of intersection of two curves or lines using technology.
SL2.5	Modelling with the following functions: • Linear models: $f(x) = mx + c$ • Quadratic models: $f(x) = ax^2 + bx + c; a \neq 0$. Axis of symmetry, vertex, zeros and roots, intercepts on the x-axis and y-axis. • Exponential growth and decay models: $f(x) = ka^x + c$ $f(x) = ka^{-x} + c$ (for $a > 0$) $f(x) = ke^{rx} + c$

	Equation of a horizontal asymptote. • Direct/inverse variation: $f(x) = ax^n, n \in \mathbb{Z}$ The y-axis as a vertical asymptote when $n < 0$. • Cubic models: $f(x) = ax^3 + bx^2 + cx + d$ • Sinusoidal models: $f(x) = a\sin(bx) + d, f(x) = a\cos(bx) + d$
SL2.6	Modelling skills: • Use the modelling process described in the "mathematical modelling" section to create, fit and use the theoretical models in section SL2.5, and their graphs. Develop and fit the model: • Given a context recognize and choose an appropriate model and possible parameters. • Determine a reasonable domain for a model. • Find the parameters of a model. Test and reflect upon the model: • Comment on the appropriateness and reasonableness of a model. • Justify the choice of a particular model, based on the shape of the data, properties of the curve and/or on the context of the situation. Use the model: • Reading, interpreting and making predictions based on the model.
AHL1.9	Laws of logarithms: $\log_a xy = \log_a x + \log_a y$ $\log_a \frac{x}{y} = \log_a x - \log_a y$ $\log_a x^m = m\log_a x$ for $a, x, y > 0$
AHL1.11	The sum of infinite geometric sequences.
AHL2.10	Scaling very large or small numbers using logarithms. Linearizing data using logarithms to determine if the data has an exponential or a power relationship using best fit straight lines to determine parameters. Interpretation of log-log and semi-log graphs.
AHL2.7	Composite functions in context. The notation $(f \circ g)(x) = f(g(x))$ Inverse function f^{-1}, including domain restriction. Finding an inverse function.
AHL2.8	Transformations of graphs. Translations: $y = f(x) + b$; $y = f(x - a)$ Reflections: in the x-axis $y = -f(x)$, and in the y-axis $y = f(-x)$

	Vertical stretch with scale factor p: $y = pf(x)$ Horizontal stretch with scale factor $\frac{1}{q}$: $y = f(qx)$ Composite transformations.
AHL 2.9	Exponential models to calculate half-life. Natural logarithmic models: $f(x) = a + b\ln x$ Sinusoidal models: $f(x) = a\sin\left(b(x - c)\right) + d$ Logistic models: $f(x) = \frac{L}{1+Ce^{-kx}}; L, C, e > 0$ Piecewise models.

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8 Modelling periodic phenomena: trigonometric functions and complex numbers

Syllabus reference	Syllabus content
SL2.5	Modelling with the following functions: • Linear models: $f(x) = mx + c$ • Quadratic models: $f(x) = ax^2 + bx + c; a \neq 0$. Axis of symmetry, vertex, zeros and roots, intercepts on the x-axis and y-axis. • Exponential growth and decay models: $f(x) = ka^x + c$ $f(x) = ka^{-x} + c$ (for $a > 0$) $f(x) = ke^{rx} + c$ Equation of a horizontal asymptote. • Direct/inverse variation: $f(x) = ax^n, n \in \mathbb{Z}$ The y-axis as a vertical asymptote when $n < 0$. • Cubic models: $f(x) = ax^3 + bx^2 + cx + d$ • Sinusoidal models: $f(x) = a\sin(bx) + d, f(x) = a\cos(bx) + d$
SL2.6	Modelling skills: • Use the modelling process described in the "mathematical modelling" section to create, fit and use the theoretical models in section SL2.5, and their graphs. • Develop and fit the model: • Given a context recognize and choose an appropriate model and possible parameters. • Determine a reasonable domain for a model. • Find the parameters of a model. Test and reflect upon the model: • Comment on the appropriateness and reasonableness of a model. • Justify the choice of a particular model, based on the shape of the data, properties of the curve and/or on the context of the situation. Use the model: Reading, interpreting and making predictions based on the model.
AHL2.9	In addition to the models covered in the SL content the HL content extends this to include modelling with the following functions: • Exponential models to calculate half-life. • Natural logarithmic functions: $f(x) = a + b\ln x$ • Sinusoidal models: $f(x) = a\sin(b(x - c)) + d$

	• Logistic models: $f(x) = \frac{L}{1+Ce^{-kx}}$; $L, C, k > 0$ • Piecewise models.
AHL3.8	The definitions of $\cos\theta$ and $\sin\theta$ in terms of the unit circle. The Pythagorean identity: $\cos^2\theta + \sin^2\theta = 1$ Definition of $\tan\theta$ as $\frac{\sin\theta}{\cos\theta}$ Extension of the sine rule to the ambiguous case. Graphical methods of solving trigonometric equations in a finite interval.
AHL2.8	Transformations of graphs. Translations: $y = f(x) + b$; $y = f(x - a)$ Reflections: in the x-axis $y = -f(x)$, and in the y-axis $y = f(-x)$ Vertical stretch with scale factor p: $y = pf(x)$ Horizontal stretch with scale factor $\frac{1}{q}$: $y = f(qx)$ Composite transformations.
AHL1.12	Complex numbers: the number i such that $i^2 = -1$. Cartesian form: $z = a + bi$; the terms real part, imaginary part, conjugate, modulus and argument. Calculate sums, differences, products, quotients, by hand and with technology. Calculating powers of complex numbers, in Cartesian form, with technology. The complex plane. Complex numbers as solutions to quadratic equations of the form $ax^2 + bx + c = 0$, $a \neq 0$, with real coefficients where $b^2 - 4ac < 0$
AHL1.13	Modulus–argument (polar) form: $z = r(\cos\theta + \mathrm{i}\sin\theta) = r\mathrm{cis}\,\theta$ Exponential form: $z = re^{\mathrm{i}\theta}$ Conversion between Cartesian, polar and exponential forms, by hand and with technology. Calculate products, quotients and integer powers in polar or exponential forms. Adding sinusoidal functions with the same frequencies but different phase shift angles. Geometric interpretation of complex numbers
AHL3.7	The definition of a radian and conversion between degrees and radians. Using radians to calculate area of sector, length of arc.

9 Modelling with matrices: storing and analysing data

Syllabus reference	Syllabus content
AHL1.14	Definition of a matrix: the terms element, row, column and order for $m \times n$ matrices. Algebra of matrices: equality; addition; subtraction; multiplication by a scalar for $m \times n$ matrices Multiplication of matrices. Properties of matrix multiplication: associativity, distributivity and non-commutativity. Identity and zero matrices. Determinants and inverses of $n \times n$ matrices with technology, and by hand for 2×2 matrices. Awareness that a system of linear equations can be written in the form $Ax = b$. Solution of the systems of equations using inverse matrix.
AHL1.15	Eigenvalues and eigenvectors. Characteristic polynomial of 2×2 matrices. Diagonalization of 2×2 matrices (restricted to the case where there are distinct real eigenvalues). Applications to powers of 2×2 matrices.
AHL3.9	Geometric transformations of points in two dimensions using matrices: reflections, horizontal and vertical stretches, enlargements, translations and rotations. Compositions of the above transformations. Geometric interpretation of the determinant of a transformation matrix.

10 Analyzing rates of change: differential calculus

Syllabus reference	Syllabus content
SL5.1*	Introduction to the concept of a limit. Derivative interpreted as gradient function and as rate of change.
SL5.2*	Increasing and decreasing functions. Graphical interpretation of $f'(x) > 0, f'(x) = 0, f'(x) < 0$.
SL5.3*	Derivative of $f(x) = ax^n$ is $f'(x) = anx^{n-1}$, $n \in \mathbb{Z}$ The derivative of functions of the form $f(x) = ax^n + bx^{n-1} + \ldots$,where all exponents are integers.
SL5.4*	Tangents and normals at a given point, and their equations.
SL5.6	Values of x where the gradient of a curve is zero. Solution of $f'(x) = 0$. Local maximum and minimum points.
SL5.7	Optimisation problems in context.
AHL5.9	The derivatives of sin x, cos x, tan x, e^x, ln x, x^n where $n \in \mathbb{Q}$. The chain rule, product rule and quotient rules. Related rates of change.
AHL5.10	The second derivative. Use of second derivative test to distinguish between a maximum and a minimum point.
AHL5.13	Kinematic problems involving displacement s, velocity v and acceleration a.

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11 Approximating irregular spaces: integration and differential equations

Syllabus reference	Syllabus content
SL5.5*	Introduction to integration as anti-differentiation of functions of the form $f(x) = ax^n + bx^{n-1} + \ldots$, where $n \in \mathbb{Z}, n \neq -1$ Definite integrals using technology. Anti-differentiation with a boundary condition to determine the constant term. Areas between a curve $y = f(x)$ and the x-axis, where $f(x) > 0$.
SL5.8	Approximating areas using the trapezoidal rule.
AHL5.11	Definite and indefinite integration of x^n where $n \in \mathbb{Q}$, including $n = -1$, sin x, cos x, $\frac{1}{\cos^2 x}$ and e^x. Integration by inspection, or substitution of the form $\int f(g(x))g'(x)\mathrm{d}x$.
AHL5.12	Area of the region enclosed by a curve and the x or y-axes in a given interval. Volumes of revolution about the x-axis or y-axis.
AHL5.13	Kinematic problems involving displacement s, velocity v and acceleration a.
AHL5.14	Setting up a model/differential equation from a context. Solving by separation of variables.
AHL5.15	Slope fields and their diagrams.
AHL5.16	Euler's method for finding the approximate solution to first order differential equations. Numerical solution of $\frac{\mathrm{d}y}{\mathrm{d}x} = f(x, y)$. Numerical solution of the coupled system $\frac{\mathrm{d}x}{\mathrm{d}t} = f_1(x, y, t)$ and $\frac{\mathrm{d}y}{\mathrm{d}t} = f_2(x, y, t)$.

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12 Modelling motion and change in 2D and 3D: vectors and differential equations

Syllabus reference	Syllabus content
AHL3.12	Vector applications to kinematics. Modelling linear motion with constant velocity in two and three dimensions. Motion with variable velocity in two dimensions.
AHL3.13	Definition and calculation of the scalar product of two vectors. The angle between two vectors; the acute angle between two lines. Definition and calculation of the vector product of two vectors. Geometric interpretation of $\|\boldsymbol{v} \times \boldsymbol{w}\|$. Components of vectors.
AHL5.16	Euler's method for finding the approximate solution to first order differential equations. Numerical solution of $\frac{\mathrm{d}y}{\mathrm{d}x} = f(x, y)$. Numerical solution of the coupled system $\frac{\mathrm{d}x}{\mathrm{d}t} = f_1(x, y, t)$ and $\frac{\mathrm{d}y}{\mathrm{d}t} = f_2(x, y, t)$.
AHL5.17	Phase portrait for the solutions of coupled differential equations of the form: $\frac{\mathrm{d}x}{\mathrm{d}t} = ax + by$ $\frac{\mathrm{d}y}{\mathrm{d}t} = cx + dy$ Qualitative analysis of future paths for distinct, real, complex and imaginary eigenvalues. Sketching trajectories and using phase portraits to identify key features such as equilibrium points, stable populations and saddle points.
AHL5.18	Solutions of $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = f(x, \frac{\mathrm{d}x}{\mathrm{d}t}, t)$ by Euler's method.

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13 Representing multiple outcomes: random variables and probability distributions

Syllabus reference	Syllabus content
SL4.2*	Presentation of data (discrete and continuous): frequency distributions (tables). Histograms. Cumulative frequency; cumulative frequency graphs; use to find median, quartiles, percentiles, range and interquartile range (IQR). Production and understanding of box and whisker diagrams.
SL4.7*	Concept of discrete random variables and their probability distributions. Expected value (mean), $\mathrm{E}(X)$ for discrete data. Applications.
SL4.8*	Binomial distribution. Mean and variance of the binomial distribution.
SL4.9*	The normal distribution and curve. Properties of the normal distribution. Diagrammatic representation. Normal probability calculations. Inverse normal calculations.
AHL4.17	Poisson distribution, its mean and variance. Sum of two independent Poisson distributions has a Poisson distribution.
AHL4.14	Linear transformation of a single random variable. Expected value of linear combinations of n random variables. Variance of linear combinations of n independent random variables. $\bar{x}$ as an unbiased estimate of μ. s_{n-1}^2 as an unbiased estimate of σ^2.
AHL4.15	A linear combination of n independent normal random variables is normally distributed. In particular, $X \sim \mathrm{N}(\mu, \sigma^2) \Rightarrow \bar{X} \sim \mathrm{N}\left(\mu, \frac{\sigma^2}{n}\right)$ Central limit theorem.

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14 Testing for validity: Spearman's, hypothesis testing and χ2 test for independence

Syllabus reference	Syllabus content
SL4.11	Formulation of null and alternative hypotheses, H_0 and H_1. Significance levels. p-values. Expected and observed frequencies. The χ^2 test for independence: contingency tables; degrees of freedom, critical value. The χ^2 goodness of fit test.
SL4.10	Spearman's rank correlation coefficient, r_s. Awareness of the appropriateness and limitations of Pearson's product–moment correlation coefficient and Spearman's rank correlation coefficient, and the effect of outliers on each.
AHL4.12	Design of valid data collection methods, such as surveys and questionnaires. Selecting relevant variables from many variables. Choosing relevant and appropriate data to analyse. Categorising numerical data in a χ^2 table and justifying the choice of categorisation. Choosing an appropriate number of degrees of freedom when estimating parameters from data when carrying out the χ^2 goodness of fit test. Definition of reliability and validity. Reliability tests. Validity tests.
AHL4.14	Linear transformation of a single random variable. Expected value of linear combinations of n random variables. Variance of linear combinations of n independent random variables. $\bar{x}$ as an unbiased estimate of μ. s_{n-1}^2 as an unbiased estimate of σ^2.
AHL4.16	Confidence intervals for the mean of a normal population
AHL4.18	Critical values and critical regions. Test for population mean for normal distribution. Test for proportion using binomial distribution. Test for population mean using Poisson distribution. Use of technology to test the hypothesis that the population product moment correlation coefficient (ρ) is 0 for bivariate normal distributions. Type I and II errors including calculations of their probabilities.

15 Optimizing complex networks: graph theory

Syllabus reference	Syllabus content
AHL3.14	Graph theory: Graphs, vertices, edges, adjacent vertices, adjacent edges. Degree of a vertex. Simple graphs; complete graphs; weighted graphs. Directed graphs; in degree and out degree of a directed graph. Subgraphs; trees.
AHL3.15	Adjacency matrices. Walks. Number of k-length walks (or less than k-length walks) between two vertices. Weighted adjacency tables. Construction of the transition matrix for a strongly connected, undirected or directed graph.
AHL3.16	Tree and cycle algorithms with undirected graphs. Walks, trails, paths, circuits, cycles. Eulerian trails and circuits. Hamiltonian paths and cycles. Minimum spanning tree (MST) graph algorithms: • Kruskal's and Prim's algorithms for finding minimum spanning trees. • Chinese postman problem & algorithm for solution, to determine the shortest route around a weighted graph with up to 4 odd vertices, going along each edge at least once. • Travelling salesman problem to determine the Hamiltonian cycle of least weight in a weighted complete graph. • Nearest neighbour algorithm for determining an upper bound for the travelling salesman problem. • Deleted vertex algorithm for determining a lower bound for the travelling salesman problem.

**Shows content that is common to both the Mathematics: analysis and approaches and the Mathematics: applications and interpretation courses.*